**Adaptive Density Approximation**

*On the Solution of Macro Models with Distributional Channels and Default Risk*

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**ABSTRACT:** I propose a global solution method to solve heterogeneous agent models with aggregate uncertainty where distributional channels are important.

- The steady state cross-sectional distribution can be computed fairly easily in economies where all idiosyncratic uncertainty cancels out in the aggregate.
- With aggregate uncertainty, the effects do not cancel out in the aggregate, so the cross-sectional distribution changes with the stochastic aggregate shock.
- The set of state variables has to include the time-varying cross-sectional distribution of agents; with a continuum of agents, this object becomes infinite-dimensional, and thus intractable.
- The benchmark solution method in this case was developed by Krusell and Smith (1998), but it neglects distributional channels.

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**The Model**

![Diagram of the model](image)

- Active firms go to next period with chosen hours and debt, entrants choose $z$.
- New firms may enter paying a fixed entry cost.
- In adaptive sparse grids: number of points. If the function of interest exhibits sharp local behavior - discontinuities, the interpolant is small; by doing so, one is able to obtain the same accuracy with a much lower number of points.
- Classical sparse grids are constructed by leaving out subspaces whose contribution to the polynomial is not significant. The algorithm iteratively refines the grid to achieve a desired accuracy.
- Adaptive density approximation allows for a smaller number of points, useful against curse of dimensionality.

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**Conclusions**

- Krusell and Smith is probably going to maintain its popularity due to its versatility.
- In problems where distributional channels are important alternative solution methods can reach superior performances.
- Aside from this issue, the approach proposed in this work has promising results in terms of accuracy, and being easily parallelizable it can deliver important speed gains.

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**Figure 1:** Interpolation error when both grids use 100 points.

**Figure 2:** Gain in accuracy as number of points increases for different refinement levels.

**Figure 3:** Single node performance for different levels of refinement, as number of MPI processes increase.

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**Timeline for firms**

end of $t$: Active firms go to next period with chosen hours and debt, $h_t$ and $b_t$.

beginning of $t$: $z_t$ is observed, as well as $\sigma_{z_t}$ and $\sigma_{z_t}$; $N_t$, $Y_t$, and $b_t$ are obtained from the cross-section.

- Firms that cannot cover losses by issuing new debt exit.
- New firms may enter paying a fixed entry cost $\xi$.
- Active firms choose $h_{t+1}$, $b_{t+1}$.
- Entrants choose $n_{t+1}$.

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**Cross-sectional Density Approximation**

- Algan et al. (2008): approximate the cross-sectional distribution of firms to account for distributional channels.
- This is done with a flexible functional form: moments of the distribution depend on the position in each point of the aggregate state space.
- Unfeasible in a multidimensional context.

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**Adaptive Sparse Grids**

- Classical sparse grids are constructed by leaving out subspaces whose contribution to the interpolant is small; by doing so, one is able to obtain the same accuracy with a much lower number of points.
- The function of interest exhibits sharp local behavior - discontinuities, kinks, or even steep regions; classic sparse grids will not provide accurate approximations.
- In adaptive sparse grids:
  - Interpolation is based on hierarchical piecewise polynomials with local support and varying order.
  - Basis functions are restricted to a neighborhood of each point.
  - An iterative refinement strategy is used for grid points.
- A way smaller number of points is needed: useful against curse of dimensionality!