Parallel Mesh Partitioning with Balanced $\kappa$-Means

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Parallel Scientific Computing

- Climate Simulation
- Wave Propagation
- Materials Science
- Complex Network Analysis
- ...

Balance load and Minimize Communication
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*Balance load and Minimize Communication!*
Graph partitioning problem:
Input: graph $G = (V, E)$, $k \in \mathbb{N}$ and $\epsilon \in \mathbb{R}^+$. 
Objective: Find $k$-partition of $V$ with imbalance $\leq \epsilon$ so that some objective function is optimized.
• cut weight: \( \sum_{u \in V_i, v \in V_j, i \neq j} w((u, v)) = 7 \)

• imbalance: \( \max_{V_i \in P} \left\lfloor \frac{|V_i|}{|V|/k} \right\rfloor - 1 = \left\lfloor \frac{6}{13/3} \right\rfloor - 1 = \frac{6}{5} - 1 = 0.2 \)
Two Types of Networks

Meshes
- Regular structure
- Geometry
- Similar degrees
- Large diameter

Complex Networks
- Community structure
- Hierarchy
- Heavy-tailed degree distribution
- Small diameter
Today’s Partitioners

Geographer - for geometric meshes

- New development von Looz, Tzovas, Meyerhenke, at ICPP 2018
- Optimizes shape and implicitly communication volume

ParHIP - for complex networks

- Parallel extension of KaHIP Meyerhenke, Sanders, Schulz, in TPDS 2017
Existing Heuristics

Space-filling curves - survey by Bader, 2012

- SFC: surjective function $f : [0, 1] \rightarrow [0, 1]^d$.
- $f^{-1}$: Given a point in $[0, 1]^d$, return index $k \in [0, 1]$.
- Strategy: Compute $f^{-1}(p)$ for every point, partition points by their indices.
Existing Heuristics

Multi-Level Approach (Karypis, Kumar, 1999)

- Recursively coarsen graph
- Compute initial partition on coarsest level
- Refine solution with local search during uncoarsening (Fiduccia, Mattheyses, 1982)
# Existing Heuristics

## Others
- Recursive Bisection - Berger and Bokhari, 1987
- Diffusive Partitioning - Schamberger, 2004
- Spectral Partitioning - Hendrickson and Leland, 1995
- ...

## Speed-Quality-Tradeoff
- Multi-Level-Heuristic: Low edge cut, limited scalability
- Space-Filling Curves: Highly scalable, ragged shapes, high edge cut
- Diffusive Partitioning: Convex shapes, high quality, very slow
Geographer
Balanced $k$-means for Parallel Geometric Partitioning
von Looz, Tzovas, Meyerhenke 2018
International Conference on Parallel Processing (ICPP)
**k-Means algorithm**

Lloyd, 1982

- Classical $k$-Means: Well-known geometric clustering
- Alternate steps:
  - Assign each point to closest cluster
  - Move each cluster center to mean of assigned points
$k$-Means algorithm
Lloyd, 1982

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- Alternate steps:
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- Optimizes $\sum_{i=1}^{n} d(i, c_i)^2$, sum of squared distances to cluster centers
- Guaranteed to converge to local minimum
- Easy to parallelize
**$k$-Means algorithm**

Lloyd, 1982

- Classical $k$-Means: Well-known geometric clustering
- Alternate steps:
  - Assign each point to closest cluster
  - Move each cluster center to mean of assigned points
- Optimizes $\sum_i^n d(i, c_i)^2$, sum of squared distances to cluster centers
- Guaranteed to converge to local minimum
- Easy to parallelize
- No balance
- Convergence highly dependent on starting positions
Balanced $k$-Means

**Approach**

- Add *influence* value to each block
- In assignment step, assign each point $i$ to cluster $c_i$ with smallest $d(i, c_i)^2 / \text{influence}(c_i)$
- Adjust influence values to match required block sizes

**Influence Adjustment**

- Volume of $d$-dimensional hypersphere scales $\propto \text{radius}^d$
- Let $\gamma(c)$ be ratio of current and target size of cluster $c$
- Then: $\text{influence}[c] \leftarrow \text{influence}[c]/\gamma(c)^{1/d}$.
Combined Algorithm

**Geographer** for *Geometric and graph-based partitioner*

1. Sort and redistribute input points with space-filling curve
2. Select $k$-Means centers equidistant on space-filling curve
3. Use distributed, balanced $k$-Means to get partition
4. Optional: Refinement with Multi-Level heuristic
   - Build graph hierarchy with matching-based coarsening
   - Refine with Fiduccia-Mattheyses during uncoarsening
Experimental Setup

Partitioners

- Geographer - (only geometric phase)
- RCB - Recursive Coordinate Bisection
- RIB - Recursive Inertial Bisection
- MultiJagged - Recursive Multisection Deveci et al., 2016
- HSFC - Space-filling Curves
Experimental Setup

Datasets

- Weak scaling: Graphs of Delaunay triangulations, $2^5$ to $2^{13}$ processes, 12 million to 1.6 billion vertices.
- Quality comparison: Synthetic and real simulation meshes

Machine

- SuperMUC petascale system
- 32 GB RAM, two Intel Xeon E5-2680 per node
Weak scaling on Delaunay graphs
(a) DIMACS graphs (2D)  
(b) Climate graphs (2.5D)  
(c) Alya and Delaunay (3D)
ParHIP
Parallel Graph Partitioning for Complex Networks
Meyerhenke, Sanders and Schulz 2017
IEEE Transactions on Parallel and Distributed Systems (TPDS)
Basic Idea

- aggressive contraction / simple and fast local search
- main idea: contract *clusterings*
- clustering paradigm: *internally dense and externally sparse*
Basic Idea

Contraction of Clusterings

- contraction: respect balance and cut
- avoid large blocks: size constraint $U$
- recurse until graph is small
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• with 512 processes, uk-2007 partitioned in 15.2 seconds (seq. 10.5min)
• 72 seconds for random geometric graph with $\approx 22G$ edges
Conclusion

Geographer - space-filling curves and balanced $k$-means

- Scales to thousands of processes, billions of vertices
- 10-20% average reduction in communication volume
- 4-14% average reduction in SpMV communication time

ParHIP - label propagation and evolutionary partitioning

- Using cluster contraction for fast, effective coarsening
- Scalable on complex networks
- High quality as measured by edge cut
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Thanks - People

• **Michael Axtmann** - for a preliminary implementation of his scalable sorting algorithm
• **Vadym Aizinger** - for supplying climate simulation meshes
• **Christian Schulz** - for supplying presentation material on ParHIP
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You - for listening!