Simulating the Solid Earth and Planets over Billions of Years: From Magma Oceans to Plate Tectonics to Exoplanets

Paul J. Tackley, ETH Zürich
The solid Earth is dynamic!

- Volcanos, earthquakes, mountains, continental drift, ...
- Wide range of timescales (seconds...billions years)
Convection is the key process
Here focus on the solid mantle

- Heat sources: radioactive heating, planetary cooling
- The oceanic plates are part of this convection
The Earth system: everything is coupled

- Atmosphere
- Life
- Plate tectonics and volcanism
- Hydrosphere
- Mantle convection
- Core convection
- Geomagnetism

Stevenson 2007
The Earth system:

everything is coupled

Stevenson 2007
Modelling Challenges

• Multi-scale problem
  – Length: mm to 1000s km
  – Time: seconds to billions of years

• Resolution: no limit to what is needed!

• Rheology
  – Large temperature-dependence (~40+ orders of magnitude)
  – Nonlinear
  – Brittle failure & plasticity
  – Elasticity
Dynamical lengthscales

Global

‘Human’ scale

1 Schematic diagram showing the processes that occur in the mantle. The lithosphere – the outermost layer of the Earth – is made up of tectonic plates that move relative to one another. Where two plates converge, the heavy oceanic plates (blue) sink into the mantle in a process known as subduction, which cools the mantle below. Continental plates (green), which are lighter, do not subduct – at the boundaries between these plates earthquakes and volcanoes occur, and mountain ranges are formed. Hot material rises from the base of the mantle in the form of “plumes”, causing volcanoes to form.
Compositional lengthscales

Outcrops of mantle rocks:

~cm

Global (mantle interior)

Trace element variations in erupted basalts

Allegre & Turcotte 1982

Deschamps, Trampert, Tackley (2005)

R.S. White
Modelling Challenges

• Multi-scale problem
  – Length: mm to 1000s km
  – Time: seconds to billions of years

• Resolution: no limit to what is needed!

• Rheology
  – Large temperature-dependence (~40+ orders of magnitude)
  – Nonlinear
  – Brittle failure & plasticity
  – Elasticity
Need huge number of grid points / cells / elements!

- e.g., to fill mantle volume:
  - $(8 \text{ km})^3$ cells (oceanic crust) $\rightarrow$ 1.9 billion cells
  - $(2 \text{ km})^3$ cells $\rightarrow$ 123 billion cells
Challenges

• Multi-scale problem
  – Length: mm to 1000s km
  – Time: seconds to billions of years

• Resolution: no limit to what is needed!

• Rheology
  – Large temperature-dependence (~40+ orders of magnitude)
  – Nonlinear
  – Brittle failure & plasticity
  – Elasticity
Equations: simplest version

- Boussinesq, infinite Prandtl number

\[ \nabla \cdot \left( \eta \left( v_{i,j} + v_{j,i} \right) \right) - \nabla P = Ra \cdot T^\wedge \]

\[ \eta_{eff} = \min \left[ \eta(T), \frac{\sigma_{yield}}{2\dot{e}} \right] \nabla \cdot \vec{v} = 0 \]

\[ \frac{\partial T}{\partial t} = \kappa \nabla^2 T - \vec{v} \cdot \nabla T + H \]
The rheological challenge

- Viscous, T-dependent rheology appropriate for the mantle:

\[ \exp\left(\frac{E}{kT}\right) \quad \text{where } E\sim340 \text{ kJ/mol} \]

T from 1600 \( \rightarrow \) 300 K

\[ \Rightarrow 1.3\times10^{48} \text{ variation} \]

- Non-Newtonian (power-law), plastic, elastic, brittle deformation also important in some areas
Talk Plan

• Introduction
• Numerical method & parallel scaling
• Application to understand plate tectonics
Numerical Method

Lagrangian tracers (markers) for composition

Figure by T. Gerya
Staggered grid (finite volume) discretization

- If orthogonal -> easy finite differences
- All derivatives involve adjacent points
- Avoids checkerboard pressure solution
Finite volume version of the x-momentum equation

\[
- \frac{p_{IJ} - p_{I-1J}}{\Delta x} + \frac{\sigma_{xx,IJ} - \sigma_{xx,I-1J}}{\Delta x} + \frac{\sigma_{xz,ij+1} - \sigma_{xz,ij}}{\Delta z} = 0
\]
Similarly for z-momentum and mass conservation
Solvers

- Either geometric multigrid or
- direct solvers via PETSc
Stag3D: 1992-

1993 GRL

1998 AGU monograph

Intel

Touchstone

Delta

Spherical??
‘Yin-Yang’ spherical grid
(Kageyama & Sato 2004 G$^3$)

- Orthogonal => finite-differences possible
- Small overlap (minimum overlap version)
‘Yin-Yang’ spherical grid (Kageyama & Sato 2004 G$^3$)

- Orthogonal $\Rightarrow$ finite-differences possible
- Small overlap (minimum overlap version)
Avalanches in the mantle

1993: supercomputer, spectral code

15 years of progress

2008: laptop, multigrid code
Iterations on up to 8192 cores of Cray XT5

Monte Rosa @ CSCS
Nodes: 2 x 6-core Opteron

Communication overhead becomes dominant at small #points/core
Multigrid V cycle

Smooth 32x32x32
Smooth 16x16x16
Smooth 8x8x8
Exact solution 4x4x4

Residues (=error)
Multigrid: Optimized
Resulting scaling (Multigrid, yin-yang)

Cray XT5

1.8 billion
Scaling model, 8* azimuthal decomposition: Multigrid

Multigrid – prediction

Time (s)

#cores

4096x12288x4096x2 (412G)
2048x6144x2048x2 (51G)
1024x3072x1024x2 (6.4G)
512x1536x512x2 (800M)
256x768x256x2 (100M)
128x768x128x2 (12.6M)
Scaling model, improved decomposition: Multigrid

Multigrid – prediction

- 4096x12288x4096x2 (412G)
- 2048x6144x2048x2 (51G)
- 1024x3072x1024x2 (6.4G)
- 512x1536x512x2 (800M)
- 256x768x256x2 (100M)
- 128x768x128x2 (12.6M)
Scaling model, improved decomposition: Multigrid

#cells is enough for 2 km resl.
Scaling model, improved decomposition: Multigrid

Scaling to millions cores on large grids

#cells is enough for 2 km resl.
Summary

• Method scales to millions of cores for large enough grids
• On modern ‘most powerful in the world’ computers, could achieve 2 km global resolution
• Grid size for production runs is limited by:
  – Needing 100,000s to millions of time steps for 4.5 billion years
  – Needing 100+ cases for systematic parameter studies
• GPU kernels now implemented
StagYY physics & chemistry

• Geometry: Cartesian or spherical-shell, 3D or 2D, global or regional

• Compressible truncated anelastic approximation
• Many compositions: Harzburgite-pyrolite-basalt, primordial, continental crust,…
• Perple_X to calculate physical properties, or a simple mineralogical approach
StagYY physics & chemistry

- Rheology: Viscoelasticity, diffusion creep, dislocation creep, plasticity. ‘Laboratory’ parameters possible.
- Grain-size evolution
- Partial melting -> basaltic or felsic (continental) crust. Melt migration.
- Trace element and water tracking, including melt-solid partitioning and outgassing.
- Coupling to atmospheric evolution (water & CO2) -> changing surface temperature
- Several core cooling models (e.g. Buffett, Labrosse) -> changing CMB temperature
- Geoid calculation (self-gravitating)
- Free surface using ‘sticky air’
StagYY physics & chemistry

- Imposed plate motion history from Gplates
- Magma ocean treatment
  - Up to 100% melt
  - Mostly magma regions -> effective diffusivity
  - Melt-solid segregation
  - Radiative surface boundary
  - Melting model using oxides
Modelling examples from our research group
Other planets

Super-Earths

Icy moons

Coupled atmosphere-interior evolution (Venus)
Age of ocean floor

Digital Isochrons of the Ocean Floor

Mystery: Why a ‘Triangular’ age-area distribution?

Why? Expect a flat distribution
Supercontinent cycle

Warming under supercontinent

Rolf, Coltice, Tackley EPSL 2012
Dynamic Causes of the Relation Between Area and Age of the Ocean Floor

N. Coltice, T. Rolf, P. J. Tackley, S. Labrosse

A

Area per unit age (km² yr⁻¹)

Age (Ma)

B

- Plates
- Continent

Age (Ma)

C

- 6 continents + plumes
- 3 continents
- 1 continent

Age (Ma)
Range of plate sizes

Why? Why not a characteristic size?
Subduction controls the distribution and fragmentation of Earth’s tectonic plates, C. Mallard et al., Nature 2016
Influence of yield stress on plate distribution

Claire Mallard
Nicolas Coltice
AUGURY ERC project
Dietmar Müller

<table>
<thead>
<tr>
<th>Yield Stress = 100 MPa</th>
<th>YS = 150 MPa</th>
<th>YS = 200 MPa</th>
<th>YS = 250 MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensionless viscosity</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
</tr>
<tr>
<td>Dimensionless</td>
<td>(e)</td>
<td>(f)</td>
<td>(g)</td>
</tr>
<tr>
<td>temperature</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Higher yield stress -> larger plates
Generate plate maps

Plates are not a uniform size
-> a distribution from large to small
Plate size distribution

YS=200 MPa -> matches Earth!
High curvature => new small plates form
Large plates stay ~same
How/when did continents form?
Continental crust formation on early Earth controlled by intrusive magmatism

A. B. Rozel\(^1\), G. J. Golabek\(^2\), C. Jain\(^1\), P. J. Tackley\(^1\) & T. Gerya\(^1\)

Understanding continental crust formation

LETTER Nature 545, 332-335 (2017)

doi:10.1038/nature22042

- How & when continental crust formed is controversial.
- Here we show that, surprisingly, it could have formed before plate tectonics (subduction) started.
- Predicted volcanism and tectonic styles match geological observations.

Computational Model

Comparison with Observations
Outlook
- PASC Software Development Project
- Staggered Grid Base Layer
- High performance, scalable operations on staggered finite volume/finite difference grids for geodynamics
- Parallel C/MPI Library (with Fortan 90 interface)
- Accelerate 3 target application codes: StagYY, I3ELVIS, and LaMEM
- Includes a self-contained demonstration code (StagBLDemo) for regional geodynamics
- PETSc interface for Staggered Grid Abstraction (DMStag)
- Default Settings, Documentation, and Examples as first-class deliverables
- Support adaptive mesh refinement AMR
- Provide multigrid convergence analysis tools
- Operator kernels with GPU-optimized backends