Challenges in the first principle modelling of Magneto Hydro Dynamic (MHD) instabilities and their control in magnetic fusion devices.

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Outline:

1. Introduction
   • Fusion: clean and safe energy
   • Magnetic confinement of fusion plasmas.
   • ITER – international next-step fusion project.

2. Important role of the first principle modelling and HPC for magnetic fusion research.
   • MHD control is essential in magnetic fusion devices and in ITER in particular.
   • Progress in understanding of physics of Edge Localized Modes and their control done with European non-linear MHD code JOREK (http://jorek.eu/)

3. Conclusions.
Fission reactions are already the source of controlled energy. Fusion is more efficient, but plasma is more difficult to confine and control. Experimental stage now.
Our sun is not very efficient fusion reactor…

Hydrogen $\rightarrow$ Helium

$\text{mW/kg}$

$\text{W/kg}$
How hot it should be to overcome Coulomb repulsion of ions? What fuel for fusion reactors?

\[ P \left( \frac{W}{m^3} \right) = n_1 n_2 \text{reactivity } E_{\text{fusion}} \]

\[ D + T \rightarrow {}^4\text{He}[3.5 \text{ MeV}] + n[14.1 \text{ MeV}] \]

\[ E = \Delta mc^2 \]

\[ D + D \] fuel in present day experiments

\[ T_{DD} \approx 150 \text{ millions K} \]

\[ \approx 15 \text{ keV} \]

\[ H = \rightarrow {}^4\text{He} \approx 15 \text{ millions degrees} \]

Sun core

\[ \approx 15 \text{ millions degrees} \]

JET(UK)

\[ D + T \] fuel for ITER and future reactors

\[ T_{DT} \approx 200 \text{ millions K} \]

\( (20 \text{ keV}) \)

D+T experiment on JET(UK) 1992

M. Becoulet, PASC18, Basel, Switzerland
D-T fusion: clean energy which can last billion years

\[ D + T \rightarrow ^4\text{He}(3.5\text{MeV}) + n(14\text{MeV}) \]

\[ ^6\text{Li} + n \rightarrow ^4\text{He} + T \]

\[ D + ^6\text{Li} \rightarrow 2^4\text{He} \]

Fuels of D-T fusion: deuterium / lithium
Ashes of D-T fusion: hélium
Deuterium 33g/m$^3$ water: billions of years
Lithium (for T): 0.17g/m$^3$ sea water: millions of years
Magnetic confinement. Tokamak principle.

Particles cyclotron motion + drifts due to gradients.

\[ \rho_i = \frac{m_i v_{\perp}}{e B} \approx 10^{-3} \text{ m} \]

ion Larmor radius
Sheared magnetic lines belong to the corresponding magnetic surfaces.

\[ j \times B = \text{grad } P \]

\[ B \cdot \text{grad } P = 0 \]
Magnetic field works like thermal insulator. Parallel to magnetic lines heat diffusivity / perpendicular:

\[ \frac{\chi_{||}}{\chi_{\perp}} \sim 10^{11} \]

- \( T_{\text{center}} \sim 10 - 20 \text{keV} > 10 \) times \( T_{\text{Sun core}} \)
- \( T_{\text{edge}} \sim 100 \text{eV} \)
- \( T_{\text{wall}} \sim 0.03 \text{eV} \)
- \( n_e \sim 10^{20} \text{m}^{-3} \)
- 10^{-6} of atmosphere
- only few g of plasma
- Power fluxes: 10-20MW/m²
Not only tokamaks! Other magnetic fusion device - stellarator (no plasma current). Complicated magnetic coils.

Large Helical Device (LHD) Nagoya Japon

No plasma current => more MHD stable, but not very « good » flux surfaces => usually confinement is lower compared to tokamaks. Next step = ITER will be a tokamak.
From present day tokamaks to power plants: plasma confinement increases with size 😊 (but the cost also 😞)

\[ Q = \frac{P_{\text{fusion}}}{P_{\text{input}}} \]

- amplification factor

\[ nT\tau_E > 5 \times 10^{21} \text{[m}^3\text{keV s]} \]

- ignition condition for D-T (Lawson)

\[ \tau_E = \frac{W_{\text{th,plas}}}{P_{\text{loss}}} \]

- confinement time increases with size \( \sim I_p R^2 \)

\( \text{DIII-D(US)} \) + many tokamaks around the world

WEST(France)

- 25 m\(^3\)
- \( P_{\text{fusion}} \sim 0 \text{ MW} \)
- \( Q < 1 \)

JET(UK)

- 80 m\(^3\)
- \( \sim 16 \text{ MW} \)
- \( Q = 0.5 \)

ITER(construction France)

- 830 m\(^3\)
- \( \sim 500 \text{ MW} \)
- \( Q > 10 \)

DEMO

- \( \sim 1500 - 2000 \text{ m}^3 \)
- \( \sim 4500 \text{ MW} \)
- \( Q > 30 \)
Heating, and fuelling are needed for fusion devices to work. Fusion is safe: if no heating reactions simply stop (no chain reactions!).

To overcome Coulomb repulsion of ions plasma should be HOT.

Radio Frequency (RF) waves:
- heating, current drive

Ion Cyclotron RH (~50 MHz)

Neutral Beam Injection (NBI):
- heating, fuelling, current drive

Low Hybrid (~few GHz)
Goals in physics:
- to prove that magnetic fusion can be large-scale and carbon free source of energy, achieve Q>10;
- self heating by \( \alpha \)-particles;
- plasma control;

Goals in technology:
- materials at high energy and particle fluxes;
- plasma heating systems;
- test technologies for a next step fusion power plant

Cost: \( \sim \$20 \text{ billion} \) (Soccer World Cup in Russia \( \sim \$12 \text{billion}, \)
in Qatar\( \sim \$220 \text{billion} \))

Agreement signature: 2006
First plasma: 2025

ITER - fusion experimental reactor, a major international research project involving the EU, China, India, Japan, the Russian Federation, South Korea and the United States.

https://www.iter.org

St Paul-lez-Durance, France
Fusion research: strong synergy between experiment, plasma theory, numerical modelling and computer science to guarantee the success.

Modelling is very important in fusion! Difficult: extreme conditions, very complex geometry, electro-magnetic fields, turbulent transport, MHD instabilities, large variation in space (0.1mm-few m) and time (10^{-6}s-1000s) scales.

1. Achieve maximum confinement (=performance!) => understand and minimize heat and particle transport (=turbulence) in tokamaks.

2. Equilibrium, MHD stability, safe and high confinement scenarios, plasma control (this talk)


4. Efficient Heating and current drive.
First principles (turbulence, MHD, RF, NBI heating, etc...) : fully self-consistent calculations, numerically very heavy (very large range of time and space scales, 6D, 5D+time, 3D+time). HPC

Integrated modelling (scenarios) Realistic geometry, plasma-wall interaction, heating,… but simplified transport and heating models (coming from first principle modelling or experiment) => 1D, “1and1/2 D” (averaged over magnetic surface). time ~ seconds, minutes.

MHD instabilities => limits

Edge plasma
Radiation, recycling

Core plasma
Transport of particles, heat, current, momentum

Fusion reactions
α-heating

Plasma facing components
Heat load, erosion

Sources
Particles, heat, current, momentum

Equilibrium

M. Becoulet, PASC18, Basel, Switzerland
First principle modelling is challenging: 3D complex geometry, large range of space ($10^{-3}$m to m), time (Alfven $10^{-7}$s to min) and energy (background+ fast particles) scales. HPC challenges are extreme!

**Particle description.** Motion in macroscopic fields (Maxwell) including external. 6DxN degree of freedom; N~$10^{23}$ particles in ITER! Is used for “test” particles (fast, impurities)

**Kinetic description.** Distribution function: $f_s(X,V,t)$ probability to find a particle of specie "s" in volume $dX$ $dV$ around a point $(X,V)$. 6D+time. Gyrokinetic-5D+time. Used in turbulence modelling.

**Fluid description.** Equations for the moments of the distribution function (contain higher orders of moments= “closure” of equations?). **Hybrid** –coupled with kinetic or/and particle approach.
- **L-mode**: low confinement, basic plasma, turbulence everywhere

- **H-mode**: high confinement (foreseen for ITER), low turbulent transport at the edge, formation of “pedestal”. Steep pressure gradient => edge MHD instabilities - Edge Localized Modes (later in this talk).

- **Internal Transport Barrier**: low turbulent transport in the core, steep profiles

  Mechanisms of turbulence stabilisation (barriers)?
  Stability of these regimes?
Edge Localized Modes (ELMs) are fast (0.25ms) quasi periodic relaxations of edge profiles leading to particle and heat fluxes to plasma facing components (PFC). In ITER ELMs should be controlled!

ELMs size scaled to ITER represent an issue for ITER tungsten divertor (W) => melting, droplets ejection, cracks. “Safe” ELM if <1MJ, but predicted for ITER: 30MJ!

Similar to solar flares.

Inside tokamak JET: no ELMs

During an ELM

Plasma stream direction

divertor
**Ohm’s law**

\[ \frac{1}{R^2} \frac{\partial \psi}{\partial t} = \eta \frac{J}{R^2} - \dot{B} \cdot (\nabla \mu + \frac{\tau_{\text{C}}}{\beta} \nabla \rho) \]

**Mass density**

\[ \frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \dot{V}) + \nabla \cdot (D \nabla \rho) + S \rho \]

**Parallel momentum**

\[ \dot{B} \left[ \rho \left( \frac{\partial \dot{V}}{\partial t} + \dot{V} \cdot \nabla \dot{V} \right) - \frac{\tau_{\text{C}}}{\beta} - \nabla \times \dot{B} \right] + \nabla (\rho T) + \nabla \cdot \Pi_{\text{peo}} - \nabla \times \dot{S} - \nabla \rho - \nabla \Delta \dot{V} = 0 \]

**Poloidal momentum (vorticity)**

\[ \nabla \phi \cdot \nabla \times \left[ \rho \left( \frac{\partial \dot{V}}{\partial t} + \dot{V} \cdot \nabla \dot{V} \right) - \frac{\tau_{\text{C}}}{\beta} - \nabla \times \dot{B} \right] + \nabla (\rho T) + \nabla \cdot \Pi_{\text{peo}} - \nabla \times \dot{S} - \nabla \rho - \nabla \Delta \dot{V} = 0 \]

**Temperature**

\[ \frac{\partial (\rho T)}{\partial t} = -\dot{V} \cdot \nabla (\rho T) - \gamma \rho T \dot{V} \cdot \nabla (\dot{V} \cdot \nabla T) + \nabla \cdot \left[ \kappa \nabla T + K \nabla T \right] + (1-\gamma) S T + 0.5 \rho^2 S \rho \]

Magnetic flux aligned grid. 2D cubic Bezier finite elements: (16 control points), C1 continuity => resolution of fine structures!

Toroidal direction: Fourier harmonics.

Time stepping: fully implicit Crank-Nicholson or Gears scheme.

Preconditioned iterative method GMRES (not full matrix is inverted).

Large sparse matrix solver: (PastiX) (http://pastix.gforge.inria.fr)

HPC: MPI/OpenMP, typical run: 50.000-200.000 cpuh >20Mcpuh/year
ELMs physics was explained: peeling (current)-ballooning (pressure gradient) instabilities. ELM crash: reconnections in non-linear phase. Conductive (ergodisation) and convective (ExB) transport.

ELM filaments along magnetic field lines (observed in experiment). Filaments are cut of the main plasma in non-linear phase by sheared flows forming “blobs”. Convective density transport (ExB).

Non-linear phase: stochastic magnetic field during an ELM, reconnections. Energy conduction along perturbed field lines. Splitting of strike points in divertor (seen in experiment)

[Huijsmans NF 2013]

[Pamela PPCF 2016]
Comparison of modelling with existing experiments gives confidence in predictions for ITER. Large heat transient heat fluxes in ITER! ELMs control is mandatory!

Integrated over ELM heat flux in divertor in JOREK modelling and experiment correspond well [Pamela IAEA 2016, NF 2017]
How ELMs can be controlled keeping high confinement at the same time? Different methods are studied in present day tokamaks to be used in ITER in the future.

1. Resonant Magnetic Perturbations (RMP) generated by coils can suppress/mitigate ELMs

[Resonant Magnetic Perturbations (RMP) generated by coils can suppress/mitigate ELMs.]

2. Pellet pacing. ELMs can be triggered by pellets at higher frequency reducing ELM size.

3. ELM can be triggered by vertical plasma oscillations ("kicks") at given frequency.

[Resonant Magnetic Perturbations (RMP) generated by coils can suppress/mitigate ELMs.]

[Liang PRL 2007]

[Baylor APS 2010]

[JET E. de la Luna et al]
Physics of RMPs in ELMy plasma was discovered in modelling and benchmarked on existing tokamaks. RMPs generate continuous MHD via non-linear coupling of modes => No large ELMs.

Natural ELMs | RMP only | MHD modes with RMP

[Image of plasma waves and power deposition]

ELM without RMP | with RMPs

[Becoulet PRL 2014]
Predictive modelling of RMPs in ITER helps in optimisation of RMP coils current, phasing and divertor heat fluxes.

Rotation of RMPs to uniform divertor heat flux was proposed:

Static RMPs

Rotating RMPs

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Physics of ELM triggering by pellets: non-linear MHD + pellet ablation module. Threshold in pellet size for triggering. Position of injectors (high field side of tokamak is favourable). Pellets in ITER.

Modelling for DIII-D(US) \footnote{Futatani NF2014}

- Pellet 1.3mm: no ELM triggering
- Pellet 1.8mm: ELM triggering

Pellet size: $4.0 \times 10^{21}$ D
Modelling of ELM destabilization through vertical kicks with free boundary version of JOREK (plasma, vacuum, resistive walls, coils). [Artola EPS2018, NF2018-accepted]

Edge current is induced due to a change in plasma volume during a downward kick => MHD unstable => ELM

Up: no ELM

Down: ELM

Pre-kick density

Density (only n = 6)
Much more applications are studied using JOREK code which is in constant development in physics, numerical methods, computer science (EUROFUSION Enabling Research Projects).

**Global vertical displacement event (VDE)** [Artola 2018]

**Disruptions (global loss of confinement) control by massive gas injection (MGI)**

**Runaway electrons (JOREK+ full orbit particle orbit module)** [Sommariva NF2018]

**Small ELMs regime: QH-mode** [F Liu NF2015]

**Particle modelling of impurities (W), spattering, radiation** [Van Vugt EPS2018]
Conclusions:

1. Fusion can provide unlimited source of clean and safe energy.

2. Magnetic fusion research has reached the stage of the construction of the largest experimental reactor (ITER) which should demonstrate possibility of commercial reactors based on fusion principle.

3. First principle modelling in fusion is essential for present day experiments and further progress. Strong demand of HPC resources.

4. MHD control is essential in magnetic fusion devices and in ITER in particular.

5. Significant progress in understanding of physics of Edge Localized Modes (ELMs) and their control was achieved using European non-linear MHD code JOREK.
   - Mechanism of heat and particle transport in ELM.
   - ELMs control by RMPs.
   - ELM control by pellets.
   - ELM control by kicks.
Finite elements in poloidal plane + toroidal harmonics.

2D cubic Bezier elements. In each node: \(R(s,t)\), derivatives: \(R_s, R_t, R_{st}\). C1 continuity.

- 2D cubic Bezier patch defined by 16 control points
  
  \[
  \bar{B}(s,t) = \sum_{k,n=0}^{N,N} \bar{p}_{km} \frac{N!}{k!(N-k)!} s^k (1-s)^{N-k} \frac{N!}{m!(N-m)!} t^m (1-t)^{N-m}
  \]

- C1 continuity between patches requires that the 4 boundary control points lie on a line with their neighbouring control points

- 2D Discretisation of poloidal plane:
  - \(i=1:4\) vertices of one element
  - \(j=1:4\) basis functions at each vertex

Linearised Crank Nicholson scheme (or Gear’s scheme):

\[
\frac{\partial A(\bar{y})}{\partial t} = B(\bar{y}) \quad \Rightarrow \quad \left( \frac{\partial A(\bar{y}_n)}{\partial y} - \frac{1}{2} \delta t \frac{\partial B(\bar{y}_n)}{\partial y} \right) \delta \bar{y} = B(\bar{y}_n) \delta t
\]

Time stepping.

Master Finite Element

- local coordinate system \((s,t)\)
- local node number
- index of the side of the element

\[
R(s,t) = \sum_{i=1}^{4} \sum_{j=1}^{4} R_{ij} h_{ij} H_{ij}(s,t)
\]

\[
Z(s,t) = \sum_{i=1}^{4} \sum_{j=1}^{4} Z_{ij} h_{ij} H_{ij}(s,t)
\]

O. Czarny, JCP 2008

M. Becoulet, PASC18, Basel, Switzerland
Main problems & limits: go to realistic resistivity (ITER S~$10^9$-$10^{10}$, in JOREK S<$10^8$). Preconditioning GMRES is physics based (independent harmonics), but if highly non-linear GMRES could diverge.

- **Matrix solution:**
  - Preconditioned GMRES:
    - Use sub-matrices of each toroidal harmonic as preconditioner
      - divides factorisation of preconditioning matrix into N independent parts
        » Block-Jacobi preconditioning
    - each Factorisation and Solve parallelised using N instances of PaSTiX sparse matrix library
    - Factorisation only done when number of GMRES iterations > 20-50

- **GMRES:**
  - Matrix vector multiplication (MPI/OPENMP)
  - Matrix solve (PaStiX)

- Parallelisation scaling is challenging
  - common for implicit fluid codes